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ABSTRACT

To replicate and extend Grey's (1981, 1985) studies involving junior-high school students, "How Is Your Logic?" (a 26-item, Piagetian-based, group-administered written test of cognitive development) was given to 553 subjects, 10 through 48 years of age. Each item of the test measured either a specific concrete operation or a specific formal operation. Image analyses of the inter-item correlation matrix of the formal operations items alone completely replicated the results of a previous study, and image analyses of the inter-item correlation matrix of all the items together partially replicated the results of the same previous study. These results are interpreted as providing support for the contention that concrete operations and formal operations are defined by separate but interrelated logical processes. (Author/RH)

The Structure of Operational Thought: A Replication and Extension

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Abstract

A twenty-six item, Piagetian-based, group-administered written test of cognitive development, with each item measuring a specific concrete operation or a specific formal operation, was given to 553 subjects, ages 10-48. Image analyses of the inter-item correlation matrix of the formal operations items alone completely replicated the results of a previous study and image analyses of the inter-item correlation matrix of all the items together partially replicated the results of the same previous study. These results are interpreted as providing support for the contention that concrete operations and formal operations are defined by separate but interrelated logical processes.

The Structure of Operational Thought: A Replication and Extension

Gray (1981, 1985) suggested that the relations among various individual concrete operations and formal operations do conform to Piaget's idea of structures d'ensemble. That is, intrastage-intraoperation relations are stronger than intrastage-interoperation relations and intrastage-interoperation relations are stronger than interstage-interoperation relations. He hypothesized that such relations should be manifest in a factor analysis where each variable specifically measures one concrete operation or one formal operation. Intrastage-intraoperation relations should be represented by clearly defined factors where the underlining structure of each factor corrsponds to one concrete operation or one formal operation. Intrastage-interoperation relations and interstage-interoperation relations should be represented by the correlations among the various factors.

In presenting his argument, Gray critiqued a number of studies that focused on the concept of structures d'ensemble. In many studies, the variables did not clearly measure one concrete operation or one formal operation, thus, making it difficult to determine what was measured and eliminating any reasonable conclusion about the operations represented by the variables. Most factor analytic-based studies included relatively weak factor analytic techniques (e.g., principal components analysis) and/or consistently used an orthogonal transformation of the initial factor matrix which makes it almost impossible to investigate the relations among any

of the factors because an orthogonal transformaion automatically assumes that there is no relationship among the factors.

In contrast to previous studies, Gray (1981, 1985) used a robust factor analyatic technique (Harris image analysis, Harris, 1962) and Hofmann's (1978d) orthotran transformation procedure as well as variables that clearly represented one concrete operation or one formal operation. Using twenty-four items from two forms of a group-administered, Piagetian-based test of cognitive development, he obtained four clear formal operations factors: Make Correct Exclusions/Deny Incorrect Inclusions, Probability/Proportional Reasoning, Make a Correct Inclusion, and a generalized Combinatorial Thought Factor. These four formal operations factors were found both when the formal operations items were analyzed alone or when they were analyzed with the concrete operations items. When the concrete operations items were analzyed alone there was a Multiplication of Classes Factor and an Addition of Asymmetrical Relations Factor. When all items were analzyed together the Addition of Asymmetrical Relations Factor and three formal operations factors remained separate. The formal operations Combinatorial Thought Factor was differentiated into two separate factors, one that was mixed with the concrete operations Multiplication of Classes Factor and a separate Combinatorial Thought Factor. These results were interpreted as providing support for the idea that individual concrete operations and individual formal operations are isolatable but related to each other. Gray's data were based on 577 junior high school students which included the ages during which it is

hypothesized (Inhelder & Piaget, 1955/1958, 1959/1969; Piaget, 1947/1966, 1953/1957) that concrete operations evolve into formal operations. However, empirically (see Blasi & Hoeffel, 1974; Keating, 1980; Neimark, 1975, 1979 1982, for reviews), concrete operations evolve into formal operations over a much greater age span than that originally suggested by Piaget. The present study replicated Gray's and extended it by including subjects whose ages encompass the ages during which concrete operations empirically evolve (if they ever do evolve) into formal operations.

Method

Subjects

The original sample included 746 subjects involved in a study which focused on the relations between moral reasoning and operational thought. Mean age was 16.474 years (s = 5.195) ranging from 9.481 to 48.463 with 55.5% males and 44.5% females. Subjects were from a suburb of a medium-size midwestern city and two introductory required logic classes in the state supported university within the city. Because of scheduling problems, approximately 200 subjects did not complete both forms of the logic test. Consequently, 553 subjects (55.3% male, 44.7% female) completed both measures of operational thought and are included in the present study. Mean age of the 553 subjects was 16.662 years (s = 4.970) ranging from 10.949 to 48.463.

Procedure

The procedure and analyses follow exactly those reported by Gray (1981, 1985). Within a two week period, subjects were given

Form A and Form B, respectively, of How Is Your Logic? (Gray, 1976a, 1976b), a Piagetian-based, group-administered written test of cognitive development. Each form includes thirteen constructed response items with each item measuring a specific concrete operation or a specific formal operation. Across both forms, three concrete operations and four formal operations are assessed by a minimum of two items per operation. Responses to each item are evaluated according to the reasoning manifested in the responses. For example, a response to a formal operations item can be scored as preoperations, preoperations-concrete I, concrete I, concrete II, concrete II-formal I, formal I, or formal II, depending upon whether the item is a beginning formal operations item or a consolidated formal operations item. Similarly, responses to concrete operations items are scored as preoperations, preoperations-concrete I, or concrete I. All items are scored conservatively. That is, if a subject's response clearly indicates a concrete II level of thinking and only partially indicates a formal I level of thinking the response is scored at a concrete II level. This approach to scoring responses is in line with the Genevan approach which requires subjects to clearly demonstrate a specific operation before it can be indicated that an individual has mastered the operation.

Analyses

Scorers were doctoral students in educational psychology. All scorers were provided instruction in the scoring by the author who designed the scoring procedures. Each scorer was required to first

score 25-30 tests and the scores on the individual items were then matched with the author's scores for the items. Any discrepancies in scores were discussed and appropriate clarification of the scoring criteria were made. Subsequently, the tests that were used as training devices were returned to the subject pool and were rescored along with the remaining tests. Percent agreement for each item, where raters' scores were compared with the score given by the author, ranged from approximately 85% to 100% with only five items having an agreement score less than 100%. (All five items required scorers to evaluate sentences written by the subjects.) Coefficient alpha estimates of internal consistency were very good for the concrete and formal items together (.82) and the formal items alone (.83). The alpha for the concrete items alone was .57 and was a direct result of a ceiling effect on the items. As in the original Gray (1981, 1985) study, the Multiplication of Relations item from each form was eliminated from any analysis.

Because there is concern that correlations generated from subjects encompassing such a wide range of ages may artificially inflate the correlations (Carroll, Kohlberg, & DeVries, 1984; Humphreys & Parsons, 1979), interitem partial correlation matrices, with age partialed-out, were calculated. Next, each matrix of partial correlations was subjected to a series of Harris image analyses with the normalized initial factor matrices transformed via Hofmann's (1978d) orthotran. Average Kaiser-Meyer-Olkin measures of sampling adequacy (Kaiser, 1970) were excellent for the formal operations items and concrete operations items together (.80) and the formal operations items alone (.80), and good for

the concrete operations items alone (.68). (As with the alpha coefficients, the relatively low average measure of sampling adequacy was the result of a ceiling effect on the concrete items.) A three part approach was used to determine the number of factors to extract from each of the partial correlation matrices. The approach that was used is described in Gray (1981, 1985) and involved the use of Hofmann's (1977, 1978a) index of variable complexity, Cattell's (1966) scree test, and the psychological interpretability of the various solutions that were generated from each data set. For the formal operations items and concrete operations items together, the scree test and the psychological interpretabiity of the various solutions suggested there were four factors whereas the complexity criterion suggested there were three factors. Thus, four factors were extracted from the total set of items. Four factors were extracted from the formal operations items alone as each criterion suggested there were four factors in the formal operations data. Because the concrete operations items did not define one or more separate factors in the solutions of the concrete items and formal items together no factors were extracted from the concrete items alone.

Results[1]

The primary pattern solution for the formal items and concrete items together is presented in Table 1 and the correlations, multiple correlations, and partial correlations among the factors presented in Table 1 are presented in Table 2. The average variable

Insert Tables 1 and Table 2 about here

complexity is quite low at 1.209 and the simple structure solution of the factors is excellent with only Factor III not defined by one clear operation. Factor I represents the formal operation of Probability/Proportional Reasoning, Factor II the formal operation of Make Correct Exclusions/Deny Incorrect Inclusions, and Factor IV the formal operation of Make a Correct Inclusion. These factors are comparable to three of the six factors found in the analogous solution in Gray (1981, 1985). Specifically, Factors I, II, and IV are identical to Factors III, II, and V, respectively, reported by Gray (1981, 1985). Factor III is an integrated formal operations-concrete operations factor and is analogous to Gray's (1981, 1985) Factor I. The difference between the two solutions is that Gray's (1981, 1985) concrete operations and formal operations factor was basically a concrete operations Multiplication of Classes Factor with substantial but smaller coefficients with the beginning formal operations combination items. In the present study, the major coefficients on the integrated factor are with the formal operations combinatorial thought items and smaller but acceptable coefficients with concrete operations multiplication of classes items. The addition of asymmetrical relations items defined a separate factor in Gray (1981, 1985); in the present study, only one of the items had a coefficient greater than .300, and that was with the integrated formal operations-concrete operations factor.

The interfactor correlations and partial correlations reported in Table 2 are reasonable for such measures (Hofmann, 1978c, pp. 9-10) and, as reported by Gray (1981, 1985), the largest partial correlation is between the beginning formal operation of Make a Correct Inclusion and the consolidated formal operation of Make Correct Exclusions/Deny Incorrect Inclusions. In essence, the present data clearly replicated three operations-based factors found in an earlier study and the fourth factor is analogous to another factor found in the earlier study.

Table 3 presents the primary pattern solution for the formal items alone and Table 4 presents the correlations, multiple correlations, and partial correlations among the primary factors represented in Table 3. The factors represented in Table 3 are an

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Insert Tables 3 and 4 about here

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elegant solution. All of the factors are clearly defined by one of the four formal operations measured by the 16 items: Make Correct Exclusions/Deny Incorrect Inclusions, Probability/ Proportional Reasoning, Combinatorial Thought, and Make a Correct Inclusion, Factors I, II, III, and IV, respectively. The average variable complexity of 1.035 is outstanding and is slightly better than the 1.058 reported by Gray (1981, 1985) for the formal items. The values reported in Table 4 suggest reasonable relationships among the factors. The pattern of coefficients for the partial correlations exactly duplicates that reported by Gray for his

solution of the formal items. The operations necessary for success on the items defning each of the four factors are the same operations necessary for success on four of the Inhelder problems (Inhelder & Piaget, 1955/1958). Make Correct Exclusions/Deny Incorrect Inclusions (Factor I) and Make a Correct Inclusion (Factor IV) are the operations necessary for success at the consolidated formal operations substage and the beginning formal operations substage, respectively, on the pendulum problem; Probability/Proportional Reasoning (Factor II) is analogous to the operations necessary for success on the equilibrium in the balance problems; and Combinatorial Thought (Factor III) is analogous to the operations necessary for success on the chemical problem and the permutation problem (Piaget, & Inhelder, 1951/1975). Thus, correlations among the various traditional Piagetian tasks may be interpreted as analogous to the interfactor correlations reported in Table 4. The separate correlations between Combinatorial Thought and the two exclusion-based factors (Make a Correct Inclusion & Make Correct Exclusions/Deny Incorrect Inclusions) are similar to or greater than one reported by Kuhn, Langer, Kohlberg, and Haan (1977/, and smaller than one reported by Shayer (1979) for the relation of the combinations of colored and colorless chemical bodies problem with the pendulum problem. Similarly, the correlations between the two exclusion-based factors (I, IV) and the Probability/ Proportional Reasoning factor (II) as well as the correlation between Combinatorial Thought (III) and Probability/Proportional Reasoning (II) are comparable but smaller than those reported by Shayer (1979) for the relation of the pendulum problem with the balance problem and the chemicals problem with the balance problem, respectively.

Although these cross-study relations are suggestive, the differing levels at which the relations are occurring must be considered (i.e., zero-order correlations vs. interfactor correlations).

Also, as reported by Gray (1981, 1'85), when the concrete items are removed from the analysis, the coefficients defining the Combinatorial Thought factor (III) are all larger than when the concrete items are included in the analysis.

Because none of the concrete operations items defined a separate factor when the formal items and concrete items were analyzed together, and because of a "ceiling effect" on the concrete items which dramatically reduced the variance among the concrete items, it was decided that there was no necessity to analyze them separately. (R. J. Hofmann, Personal Communication, April 23, 1985).

Discussion

Three important points can be made about the results of the present study. First, and probably most important, is the exact replication of the results on the formal items and a partial replication of the results on the formal items and concrete items together that were reported by Gray (1981, 1985). He presented a detailed discussion of the theoretical and methodological implications of his results stressing that individual factors represent intrastage-intraoperati relations, and interfactor correlations and partial correlations represent intrastage-interoperation relations as well as interstage-interoperation relations. It was his contention that such relationships as defined by his factor analytic results clearly supported Piaget's ideas regarding the structures d'ensemble of formal operations and concrete operations. As noted previously, however, his sample was

restricted to junior high school students and, thus, did not completely cover the age span during which one would expect concrete operations to evolve into formal operations. Such a restriction of subjects' ages may have accounted for his results. If Gray's results were a function of the ages of the subjects then the greater age range included in the present study should have produced a factor structure different than the structure found in the previous study. However, the theoretically-based factor structure reported by Gray, especially with the formal operations items alone, was replicated exactly. This replication with an extended age range supports Piaget's ideas regarding the relationships among different operations that define formal operations.

A second major result revolves around the relations among the formal items and concrete items. In several factor analytic-based studies where concrete operations and formal operations have been investigated together, the concrete operations variables and the formal operations variables tend to define or are interpreted as defining one general concrete operations factor and one general formal operations factor (Guerin, 1975; Lawson, 1976, 1978; Lawson & Nordland, 1976; Lawson & Renner, 1974, 1975). Theoretically, these results are seen as supporting the separate wholistic quality (structures d'ensemble) of each operational stage. However, Piaget continually stressed that as concrete operations evolve into formal operations, concrete operations become the conter for formal operations and there is a hierachical relationship that is established between concrete operations and formal operations.

That is, concrete operations are incorporated within formal operations both as content for formal operations and as tools for use by formal operations. Such a perspective suggests that as formal operations begin to dominate thought, the impact of concrete operations lessen. Thus, a combined formal operations and concrete operations factor, where formal operations items define the factor and concrete operations items have substantial but smaller coefficients with the factor, may be interpreted as supporting such a hierarchical develoment, especially if the subject's ages encompass the ages during which concrete operations evolve into formal operations (R. J. Hofmann, personal communication, April 23, 1985).

A comparison of the integrated formal operations-concrete operations factor in Table 1 and Gray's (1981, 1985) mixed concrete operations and formal operations factor may be interpreted as supporting this position. In the present study, the formal operations combinatorial thought items have larger coefficients with the integrated formal operations-concrete operations factor than they do with the mixed concrete operations and formal oeprations factor reported by Gray (1981, 1985). Also, the coefficients of the concrete operations items with the integrated formal operations-concrete operations factor in Table 1 are smaller than with the mixed concrete operations and formal operations factor reported by Gray (1981, 1985). In fact, three out of four seriation items are so small that they are no longer considered as an item that defines a factor. Similar results with formal operations or concrete operations dominating a mixed factor have been reported, but not interpreted as such, by Lawson and Nordland (1976),

Lawson and Renner (1974, 1975), and Lee (1971), suggesting support for Piaget's ideas regarding the changing relationships between concrete operations and formal operations across the ages during which concrete operations evolve into formal operations. Obviously, only a long-term longitudinal study can adequately provide data to address this issue.

A final important result is that the content of problems used to assess operational thought is not as important as the operations needed to correctly solve the problems. Clearly, this conclusion was again supported as the items that were used were not traditional Piagetian tasks. They were written items that required the logic of formal operations or concrete operations to successfully solve the problems but the items did not include any of the content of traditional Inhelder or Piaget tasks. For example, the various exclusion items (Make Correct Exclusions/Deny Incorrect Inclusion and Make a Correct Inclusion) incorporated the logic required to solve the Inhelder pendulum problem. However, none of the content of those written exclusion items included any reference to an understanding of the functioning of a pendulum. These results with non-traditional tasks support beliefs by Roberge and Flexer (1979), Gray (1978a, 1978b), Neimark (1975) and others who have indicated that different types of problems that require operational thought to be successful need to be developed. The items on How Is Your Logic? are such a set of problems.

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Footnotes

[1]Coefficient alpha estimates of internal consistency, were computed using SPSS^X Release 2.0 for IBM VM/CMS. The Harris image analyses and orthotran transformations were computed using MUFACT (Hofmann, 1978b).

Table 1

Harris Image Analysis Primary Pattern Solution[a] for Formal Items and Concrete Items

Operation	Form & Item	Factor I	Factor II	Factor III	Factor IV	Variable Complexity[b]
Probability						
Judgment-FI	A12	700	-019	003	-003	1.001
-FII	B11	778	-077	-049	-073	1.045
Explanation	A13	841	060	-028	-042	1.017
	B12	761	000	020	064	1.015
Exclusion-Correct						
Judgment	A9	005	721	-046	-028	1.011
	B3	-084	708	022	074	1.052
Explanation	A10	018	723	-010	028	1.005
	B4	-034	737	044	037	1.107
Combinatorial Thought						
Combinations	A5	-003	-095	717	041	1.042
	B6	015	-074	671	009	1.025
Permutations	A6	189	-048	600	157	1.358
	B7	197	-121	543	211	1.705
Multiplication of Classes						
One-to-One	A7	-035	-054	512	062	1.062
	B2	-039	152	534	-118	1.288

(table continues)

Operation	Form & Item	Factor I	Factor II	Factor III	Factor IV	Variable Complexity[b]
Many-to-One	A11	089	155	465	-222	1.776
	B13	-024	084	467	-093	1.152
Seriation						
Increasing Series	A1	-097	001	292	-007	1.219
	B1	-010	-060	254	044	1.178
Decreasing Series	A4	-000	107	364	-210	1.800
	B5	007	020	152	-196	1.914
Inclusion-Correct						
Judgment	A2	076	157	-007	591	1.176
	B9	-088	-024	012	865	1.023
Explanation	A3	072	093	053	622	1.088
	B10	-021	119	024	772	1.051

Note. Probability = Probability/Proportional Reasoning operation; FI = beginning formal operations; FII = consolidated formal operations; Exclusion-Correct = Make Correct Exclusions/Deny Incorrect Inclusions aspect of the Exclusion operation; One-to-One = One-to-One Correspondence aspect of the Multiplication of Classes operation; Many-to-One = Many-to-One Correspondence aspect of the Multiplication of Classes

(table continues)

operation; Seriation = Addition of Asymmetrical Relations operation; Inclusion-Correct = Make a Correct Inclusion aspect of the Exclusion operation.

[a]Decimal points have been eliminated and coefficients greater than 300 are in italics.

[b]Average variable complexity = 1.209.

Table 2

Correlations, Multiple Correlations, and Partial Correlations Among the Primary Factors for Formal Items and Concrete Items

Factor	I	II	III	IV
I	.343	.174	.247	.300
II	.032	.433	.276	.401
III	.164	.168	.380	.304
IV	.219	.331	.176	.489

Note. Correlations in upper right, multiple correlations in main diagonal, and partial correlations in lower left.

Table 3

Harris Image Analysis Primary Pattern Solution[a] for Formal Items

Operation	Form & Item	Factor I	Factor II	Factor III	Factor IV	Variable Complexity[b]
Exclusion-Correct						
Judgment	A9	689	012	-118	017	1.061
	B3	768	-095	050	-019	1.040
Explanation	A10	751	018	-025	-015	1.004
	B4	808	-036	046	-057	1.021
Probability						
Judgment-FI	A12	-033	708	-011	022	1.005
-FII	B11	-092	802	-059	-067	1.052
Explanation	A13	051	834	006	-059	1.018
	B12	000	761	048	034	1.012
Combinatorial Thought						
Combinations	A5	-011	-084	805	-079	1.041
	B6	000	-070	772	-012	1.052
Permutations	A6	-001	100	693	069	1.062
	B7	-059	097	714	076	1.075
Inclusion-Correct						
Judgment	A2	051	056	-065	702	1.040
	B9	-077	-112	006	892	1.047

(table continues)

Operation	Form & Item	Factor I	Factor II	Factor III	Factor IV	Variable Complexity[b]
Explanation	A3	-004	048	-001	*723*	1.009
	B10	078	-043	005	*792*	1.025

Note. Exclusion-Correct = Make Correct Exclusions/Deny Incorrect Inclusions aspect of the Exclusion operation; Probability = Probability/Proportional Reasoning operation; FI = beginning formal operations; FII = consolidated formal operations; Inclusion-Correct = Make a Correct Inclusion aspect of the Exclusion operation.

[a]Decimal points have been eliminated and coefficients greater than 300 are in italics.

[b]Average variable complexity = 1.035

Table 4

Correlations, Multiple Correlations, and Partial Correlations Among the Primary Factors for Formal Items

Factor	I	II	III	IV
I	.518	.207	.301	.501
II	.011	.406	.333	.346
III	.117	.223	.464	.402
IV	.429	.217	.240	.597

Note. Correlations in upper right, multiple correlations in main diagonal, and partial correlations in lower left.